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A Proposed Analytical Model for Integrated Pick-and-Sort Systems

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Abstract

In this study we present an analytical approach for integration of order picking and sortation operations which are the most important, labour intensive and costly activity for warehouses. Main aim is to investigate order picking and sorting efficiencies under different design issues as a function of order wave size. Integrated analytical model is proposed to estimate the optimum order picking and order sortation efficiency. The model, which has been tested by simulations with different illustrative examples, calculates the optimum wave size that solves the trade-off between picking and sorting operations and makes the order picking and sortations efficiency maximum. Our model also allow system designer to predict the order picking and sorting capacity for different system configurations. This study presents an innovative approach for integrated warehouse operations.

Keywords: *Order Picking, Order Sortation, Wave Size, Warehouse, Pick-and-Sort Systems*

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1. Introduction

In supply chain performance of companies' warehouses play an important role. Increasing trend in product variety and expectation of customers for low delivery time of orders force the distribution centers for making investments. As is known, warehouses do not directly make money for companies, so distribution centers operate under constant pressure from management to reduce costs and increase efficiency. This dilemma needs to be solved for increasing customer satisfaction and decreasing the cost values of the process.

Increasing trend in product variety and rising tendency in delivering orders in a short time have put pressure on company managers to streamline efficient logistics operations. Efficiently and effectively workings of logistics network is directly related to operations in the warehouses. Basic warehouse operations are respectively like: Receiving of goods, storing of goods to shelf, picking of goods from shelf when ordered, sortation of randomly picked goods to related orders, packaging of the sorted goods and transporting. Integration between the operations are very high, they can directly affect each other's performance. So, by taking into consideration of whole system redevelopment plan must be done. Otherwise making separate improvements on operations may be beneficial locally but there cannot be a global improvement on whole system performance. For example, increasing the performance of order picking operation by a huge rate cannot directly increase the whole system performance by the same rate. If performance of the order sortation and packaging operation could not be increased by the same ratio, only a bottleneck will be seen between these operations. There will be no benefit of this local improvement for whole system.

Most of the workings done in warehouse operations focused and scope out only local problems. These types of research results are not sufficiently communicated to industry to make a significant impact on practice of warehouse operations. Some of the previous academic workings specified this problem which is noted the lack of integration between warehouse operations problems. Rouwenhorst, et al. (2000), Gu, et al. (2007), Baker and Marco (2009) and Gu, et al. (2010) are only some examples that cited note about this problem.

The purpose of this research is to provide assistance for a critical decision of optimal wave size in order picking and order sortation operations. Integration of these two operations is very important because order picking and sortation operations are the two warehouse functions that are very effective on the overall warehouse operational performance. And also cost related to the order picking operations account for more than % 50 of the total cost of a warehouse De Koster, et al. (2007).

In most of the warehouse operation systems order picking and sortation operations are separated from each other. Order picking systems are used to retrieve the products of the orders from the storage area. Randomly picked items are sent to sortation area for being sorted to related orders. In Pick & Sort systems "wave picking" method is being used. In wave picking method a group of orders is picked simultaneously with each picker being responsible for picking a single group of items for all the orders in a wave. After order picking operation finished, all of the products in order wave are put on the takeaway conveyor. Takeaway conveyor links the warehouse storing area (or order picking operation) to the order accumulation / sortation system. After all of the randomly selected products sorted and decomposed to the related orders, they are packed and sent to the shipping system by shipping conveyor. Efficient warehouse system requires a well designed and successfully integrated link of these operations.

While there are many implementation of integrated order picking and sortation in industry, academic research on integrated systems are relatively scarce. Most of the academic researchers are related with just order picking or order sortation operations.

For the sorting systems we consider, very few research has been conducted on their design and operation. Past research on sortation systems has concentrated on general accumulation and sortation system used to sort products of the orders into the appropriate sortation lane. Since, in most of the accumulation sorting system the number of orders is greater than the number of sortation lane.

First studies mentioned in here are about the operation strategies of order sortation systems. There are mainly three different operation strategy models proposed in sortation systems. First one is developed by Bozer et al (1985). They used simulation to show the advantages of loop conveyor sorting system to protect lane blocking. In this study number of sortation lanes is equal to number of the orders on the loop conveyor. But what if the number of orders is exceeds the number of sortation lane. Multiple orders can be assigned to one sortation lane. Bozer, et al. (1988) proposed an analytical model sortation strategy named as Fix Priority Rule (FPR) for lane assignment by simulating different wave size of orders. In fixed priority rules, the orders are prioritized (smallest order first and largest order first) and are assigned to the sortation lanes based on that primacy. Second one developed by Johnson, (1998) extend the study by Bozer, et al. (1988) and developed a dynamic sortation strategy named as Next Available Rule (NAR) and compare the sortation time performance of FPR and NAR. In dynamic assignment model item locations of orders in a wave are considered. Johnson (1998) suggested that assigning order when it is located on the conveyor is better than any fixed rule. Under NAR, each time an order is completely sorted, the next item to pass the bar code scanner of the associated accumulation lane defines the next order to be sorted. Third one on dynamic assignment family is developed by Eldemir and Charles (2006) named as Earliest Completion Rule (ECR). Under this rule, the next order is determined by selecting the order that has the closest last box to the accumulation lane. In other words, the order that requires minimum time to sort is selected as the next order to be sorted Eldemir and Charles (2006).

Second studies mentioned in here for sortation operations are related with design of sortation systems. In designing an order sortation system, like other subsystems, the key concept decisions include configuration parameters and operational strategies. The configuration parameters listed by Eldemir and Charles (2006) like “the number and length of induction lanes”, “induction type” (side by side induction or split induction), “type of main sortation line” (circulating or non-circulating), “length of main sortation line”, “the number of accumulation lanes”, “length of accumulation lanes”. There are three different sortation systems in literature that classified based on the number of induction and sortation lanes. One induction-one sortation (1-1), one induction-many sortation (1-M) or many induction many sortation (M-M). Karakaya (2011) compared the performance of FPR, NAR and ECR sortation models for different sortation systems by using simulation methods. Johnson and Russell (2002) proposed a different design for accumulation and induction lanes. In this model there are more than one induction and accumulation lanes. They assumed no recirculation of orders. Optimum sorting capacity of different systems could be calculated if the same product will be sent to many customers. Fedtke, et al. (2012) systematically investigates the different design alternatives for closed loop conveyor systems. Different design alternatives of closed loop tilt tray sortation conveyor systems have been investigated. Proposed model calculates the optimum number of induction and sortation lane which are important in sortation throughput. Table 1 shows the brief information about different research examples for sortation operations.

Table 1: Sortation operation research papers

Article	Method	One Induction	Many Induction	One Sortation	Many Sortation
(Bozer & Sharp, 1985)	Simulation	✓		✓	
(Bozer, et al., 1988)	Simulation	✓			✓
(Johnson&Lofgren, 1994)	Simulation	✓			✓
(Johnson, 1998)	Analytical M.	✓			✓
(Meller, 1997)	Analytical M.	✓			✓
(Schmidt&Jackman,2000)	Analytical M.	✓		✓	
(Johnson & Russel, 2002)	Analytical M.		✓		✓
(Russell & Meller, 2003)	Descriptive M.		✓		✓
(Bozer & Hsieh, 2004)	Analytical M.		✓		✓
(Eldemir, 2003)	Analytical M.	✓		✓	
(Gino , et al., 2010)	Analytical M.	✓			✓
(Karakaya, 2011)	Simulation	✓	✓	✓	✓

For the integrated order picking and sortation systems Russel and Meller (2003) developed a descriptive model that compare the manuel and automated sortation systems and shows the trade-off between picking and sorting efficieny. Parikh and Meller (2008) proposed more general framework for pick and sort systems that compares the efficieny of batch picking and zone picking strategy. A general cost model for pick and sort models is proposed as a function of number of orders in a wave and number of items in an order. The model also estimates the picking rate by taking into account travel time, picking cart capacity and travel time.

Marchet, et al. (2010) developed an analytical model to estimate the order picking efficiency as a function of wavelength. In this study, workload of the wave and overlapping effect is defined as a performance criterion. After developing analytical model in initial assessment related with order picking operations, the tradeoff between picking efficiency and sorting cost resolved by using simulation with some defined parameter values.

Koster, et al. (2010) worked on the pick and sort systems for developing the optimal number of zones needed. Increasing number of zones in a distribution center directly decrease the order picking time but increase the sortation and packing time of orders. Because sortation or packing operations can only starts when all items of order picked completely. Developed mathematical model give a chance to determine optimal number of zones by integrating the picking and sortation systems. In this working objective is to minimize the number of completed but unpacked orders by obeying the some rules related with capacity of order picking and packing operations.

In this paper we concentrate our efforts on the link between order picking and sortation operations by considering both design and operation parameter factors of warehouse. We proposed an analytical solution method to determine optimum design and operation factor values. Developed model analyze the impact of both different sorting strategies and order picking methods on the time to pick and sort a wave of orders. The reason for selecting analytical solution method is that, the problems discussed in this paper are at the operational level, which means that decisions need to be made quite frequently and the influence of these decisions is typically of a short duration and localized.

Such decisions typically need to be made quickly without extensive computational resources. This tends to encourage the use of analytical model procedures that can find a good solution reliably in a reasonable amount of time. In addition, from the management point of view, an ideal solution method should be simple, intuitive, and reliable in order to minimize the training costs in the warehouse Gu, et al. (2007).

The paper is organized as follows: in Section 2 we develop an analytical model for order picking operation. Using the model, we developed an expression for the expected order picking time at different order wave size. In Section 3 we presented an analytical model of a sortation operation. By using the analytical model of "Fixed Priority Rule" we developed an expression that shows the impact of order wave size on the expected sortation time. In Section 4, integration of the order picking and order sortation analytical model has been done. Proposed model calculates the optimum order wave size which makes the total order preparation time minimum. In section 5 we revisit our proposed models for integrated order picking and sortation operations through an illustrative example. Finally, in section 6 we summarize our results and mentioned about future works.

2. Order Picking Operation

The **order picking** or order preparation operation is one of basic operations of a warehouse.

It consists in retrieving items from the shelves and brings to front of a warehouse to satisfy customers orders. This operation is the most important and critical for warehouse performance.

In distribution centers, long lists of order are put together. Each customer order can involve different items and different number of items. In basic order picking procedure, each picker is assigned to only one order list and it is known that the products in this list are stored in different locations of the warehouse. Therefore, picker will look up every part of the warehouse in order to complete the list and will scan whether the items in the list exist or not. In this procedure, there are unnecessary transportation costs and utilization of ineffective workers.

In literature, there are several different order picking methods named as discrete order picking, zone picking, batch picking or wave picking. The reader is referred to De Koster, et al. (2007) for details of different order picking methods.

We used a different order picking method that is a combination of zone, batch and wave picking. In this method, each picker is assigned a zone and picks all items of orders stored in the assigned zone. The picker picks more than one item at a time.

The schematic layout of the system that we used is shown in Figure 1. Basically, we consider two functional areas; one is for storing area that includes products in the shelves and order picking operation done in here, one other for sortation operation. Items are randomly stored in bin-shelving storage racks.

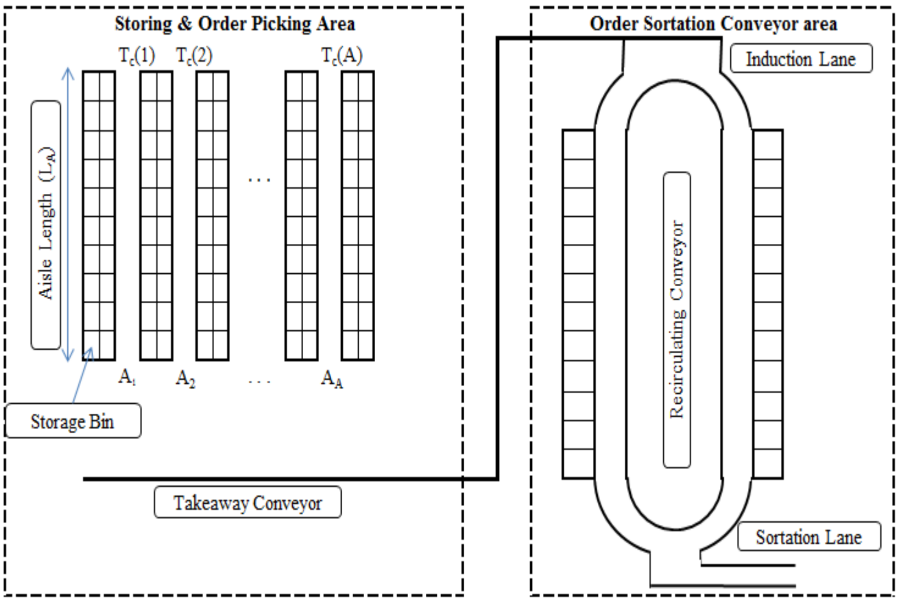

Figure 1: Top view of storing, order picking and sortation operation area

There are A aisles and in each aisle there is an order picker. Each order picker pick the products of wave orders simultaneously in different aisles. After all order picker pick all products of wave in their zone (aisle), they put them on the takeaway conveyor and transport to order sortation area.

Assumptions of analytical model of the order picking operation that described above are as below:

- There is an order picker in each aisle.
- Each of the order picker's walking speed (vp) is same.
- Picking time (Tp) of a product from each bin is same for each order picker.
- The items in the storage area are stored randomly. It is assumed that items in a wave are distributed uniformly throughout the each aisle.

Notations that we used in development of analytical model as shown in Table 2.

Table 2: Notations used in order picking models

w	Number of items within a wave, *(unit)*
A	Number of the aisle in warehouse, *(unit)*
T_p	Picking time of an item from storage bin, *(s) (Walking time for picking is not included)*
$T_c(i)$	Total picking time of all items in aisle i. *(s)*
n	Number of items that should be picked in an aisle, *(unit)*
i	Item index within an order,
j	Order index within an order,
a	Aisle index,
TPW	Total picking time of all items in a wave, *(s)*
L_A	Length of an aisle, *(m)* a=1,2,3,……,A
Rows	Number of racks on top of eachother, *(unit)*
l_u	Width of a storage bin, *(m)*
S	Number of total storage bin in warehouse, *(unit)*
v_p	Speed of order picker, *(m/s)*

2.1 Methodology of Order Picking Operation

In development of analytical models, order statistics methods are used. The classical order statistics method is defined as follow: given a sample of n random varieties X1, X2, X3, …, Xn

are the sample values placed in ascending order. They are denoted by X(1), X(2), X(3), …, X(n). The order statistics are random variables that satisfy X(1) < X(2) < X(3),< … < X(n) Anon. (2013).

For example, assume that five numbers are there in a given sample. If the sample values are like 5, 11, 2, 8, 30 and they will be showed as

X1=5, X2=11, X3=2, X4=8 and X5=30.

If we sequence the given numbers from smallest to the biggest number, they will be showed like;

X[1]=2, X[2]=5, X[3]=8, X[4]=11 and X[5]=30

Subscript [k] in x[k] indicates the k th order statistics of the sample. The minimum value in the sample named as first order statistics or smallest order statistics is X[1]=min(X1, X2, X3, …, Xn) and maximum value in the sample named as last order statistics or largest order statistics is X[n]=max(X1, X2, X3, …, Xn).

We have adapted order statistics methods to order picking operation applications.

Assume that there are n item in each aisle and picking time of all items of an order wave in aisle i is Tc(i). Order picking operation finishes after all of the items of an order wave in all aisles are picked.

Items are normally distributed in each aisle. If the location of the last item in an aisle i is determined, picking time of all items in related aisle can be calculated easily. For example items of an order wave are located in 1., 3., 5., 7. and 10. aisles. Locations of the last items in each aisle are 2., 5., 12., 4. and 20. storage bins respectively. For this example, if all of the items in 10. aisle are picked, order picking of related order wave finish.

2.1.1 Analytical Model of Order Picking Operation

Consider an order picking area shown in Figure 1 where items of an order wave are uniformally distributed in each aisle. There are n items in each of the related aisle. In each aisle there is an order picker with vp walking speed. Walking speeed is same for all order picker. Required time to go to the end of aisle (Tr) of an order picker is:

$$T_r = \frac{L_A}{v_p} \qquad (1)$$

Walking speed of all order picker is same, we can assume it as vp=1 unit so Tr is directly proportional to LA. It can be showed as:

Tr=LA (2)

If required time for order picker to go to the last item in an aisle i is denoted by **Tc(i), it will be:**

$$T_c(i) \approx normal \quad (0, T_r) \qquad (3)$$

with expected time and variance value respectively as:

$$E(T_c(i)) = \frac{T_r}{2} \quad \text{and} \quad V(T_c(i)) = \frac{T_r^{\,2}}{12} \qquad (4) \text{ and } (5)$$

Total picking time (TPW) of an order wave will be:

TPW= maximum[Tc(i)]

The general formula of probability distribution function of an order statistics is:

$$f_{x_{(k)}}(x) = \frac{n!\left(F_x(x)\right)^{(k-1)}}{(k-1)!}\frac{\left(1-F_x(x)\right)^{(n-k)}}{(n-k)!} f_x(x) \quad (6)$$

where Fx(x) is cumulative and fx(x) is probability distribution function of related distribution.

In this model we assumed that items are uniformly distributed so probability and cumulative distributions for our model respectively will be:

$$f_x(x) = \frac{1}{T_r} \quad and \quad F_x(x) = \frac{x}{T_r} \quad (7) \text{ and } (8)$$

Under order picking model, the probability distribution function for the location of the last item [n] in an aisle of the last order statistics for the uniform distribution will be:

$$f_{x_{(n)}}(x) = n\frac{x^{(n-1)}}{T_r^{\,n}} \quad (9)$$

with expected time value,

$$E[x_{(n)}] = \frac{nT_r}{n+1} \quad (10)$$

And the cumulative distribution function for the location of the last item [n] in an aisle of the last order statistics for the uniform distribution will be:

$$F_{x_{(n)}}(x) = \left(\frac{x}{T_r}\right)^n \quad (11)$$

Formula (9) and (11) shows the probability and cumulative distribution functions for the location of last item in any aisle. It is important to calculate the last item location among the whole aisle. For this, fx(n)(n) and Fx(n)(n) functions will be named as fTc(i)(n) and FTc(i)(n). By using these fTc(i)(n) and FTc(i)(n) distribution functions, the probability distribution function for the location of the last item [n] among all aisle of the last order statistics for the uniform distribution will be:

$$f_{TPW}(x) = \frac{An}{T_r^{\,An}} x^{(nA-1)} \quad (12)$$

with expected time value,

$$E(TPW) = \frac{An}{An+1} T_r \quad (13)$$

There are n items in each aisle so number of the items in an order wave (w) will be:

w=An (14)

so,

$$E(TPW) = \frac{w}{w+1} T_r \quad (15)$$

Length of an aisle (LA) can be formulated as:

$$L_A = \frac{Sl_u}{2.A.Rows} \tag{16}$$

By putting formula (16) into the formula (1) required time to go to the end of aisle (Tr) of an order picker will be:

$$T_r = \frac{Sl_u}{2.A.Rows\ .v_p} \tag{17}$$

Required time to pick all items of an order wave consists walking time to the last items location (calculated in E(TPW)) and picking time of all items from the storage bins.

So general formula for picking all items in an order wave will be:

$$T_{OrderPicking} = \frac{wT_r}{w+1} + \frac{w}{A} T_p = \frac{w}{(w+1)} \frac{Sl_u}{2.A.Rows\ .v_p} + \frac{w}{A} T_p \tag{18}$$

Where S, lu, A, Rows, Tp and vp parameters values are known. So order picking time of an order wave is dependent on only the number of the items in a wave (w) parameter.

3. Order Sortation Models

In an automated order sortation system, sorters accommodate mixed orders or batches to its own shipment destinations at a time interval. The sorting process involves identifying the item's destination, tracking the item along its conveyor path and then physically transmitting the carton or packages to the appropriate destination.

Picked products of order wave are transferred to induction area by takeaway conveyor and than all of the products of wave are sequentially enters the sortation conveyor system. Before entering into the sortation conveyor system, a barcode scanner reads the items and determine the sequence of the items of different orders. Determination of this sequence is very important. Because this sequence information is being transferred to the computer system that is controlling the sorter. Later, the computer will signal the system to divert the item into the appropriate shipment destination for packing. Top view of the recirculation sorter is shown in Figure 1. In our model there is one induction and one sortation bin (1-1 model).

Major parameters for sortation operation are the "length and speed of conveyors", "the wave size", "number of the sorting lanes" and the "sorting strategy".

Different analytical models are available for automated sortation strategies (FPR, NAR or ECR). In this part, we will mention about analytical model of FPR.

Order statistics models are used for FPR sortation analytical model. After all of the picked items of a wave are put on the recirculation conveyor, if location of the first item and last item of an order between all items on the conveyor could be determined then sortation time of that order wave could be calculated by using order statistics method. Used notations in this study have been listed before in Eldemir & Charles (2006). Table 3 shows the information about used notations:

Table 3: Notations used in order sortation models

w	Number of items within a wave, *(unit)*
y	number of items within an order, *(unit)*
m	number of orders within a wave, *(unit)*
L	Length of the closed-loop conveyor, *(m)*
v_c	Speed of the conveyors, *(m/s)*
t	The time for an item to circulate around the main sortation line, *(s)*
i	item index within an order
j	order index within a wave

3.1 Sortation Time for FPR

We begin by showing the case of sortation model, Fixed Priority Rule (FPR). FPR model commonly use rules as sort the largest or smallest orders first. In this model the sortation time of an order wave is same regardless of how orders are located on the recirculation conveyor.

Consider a recirculation conveyor shown in Figure 1, where each items sequence information are determined by a scanner that located at the entrance of the accumulation point. The FPR sortation method works as follows. First, the number of accumulation and sortation lanes is considered one and the number of items in an order is assumed fixed (y).

The length of the recirculation conveyor is:

$$T = \frac{L}{v_c} \tag{19}$$

Assumed that the location of item [i] (lji) of order [j] is uniformly distributed on the conveyor (lji ~ Uniform [0,T]) when order accumulation and sortation system starts sorting order [j]. Therefore, the location of the items that belong to order [j] will be ordered statistics. Relative positions of items on the conveyor will not change throughout the sortation process because of the constant speed of conveyor. Therefore, the time distance between the first and last items of an order (spread) can be viewed as the difference between the first and the last order statistics. Eldemir & Charles (2006)

We call the item in order j that is closest to the scanner as the first item [1] in the order and the farthest from the scanner as the last item [y]. Under FPR, the probability distribution function the order first order statistic for the uniform distribution (distribution of the first item location in order j) will be:

$$f_{l_{[j][1]}}(l \mid FPR) = \begin{cases} \left(\frac{y}{T}\right)\left(1 - \frac{l}{T}\right)^{y-1} & 0 \le l \le T; \\ 0 & otherwise \quad ; \end{cases} \tag{20}$$

with expected time value,

$$E(l_{j[1]} \mid FPR) = \frac{T}{y+1} \tag{21}$$

where $E(l_{j[1]} \mid FPR)$ refers to the location of the first item in order j using the FPR method.

Likewise the probability distribution function for the location of the last item [y] in order j of the last order statistic for the uniform distribution will be:

$$f_{l_{[j][y]}}(l \mid FPR) = \begin{cases} \left(\frac{y}{T}\right)\left(\frac{l}{T}\right)^{y-1} & 0 \le l \le T; \\ 0 & otherwise \quad ; \end{cases} \tag{22}$$

The expected time for location of the last item will be:

$$E(l_{j[y]} \mid FPR) = \left(\frac{y.T}{y+1}\right) \tag{23}$$

Therefore the expected time difference between the last item and first item can be determined by calculating the difference of their expected means as below:

$$E(difference \mid FPR)_j = \left(\frac{y.T}{y+1}\right) - \left(\frac{T}{y+1}\right), \\ = \frac{T(y-1)}{y+1} \tag{24}$$

Under FPR, the orders are ranked at the beginning of the sortation process. Since the location of the items in each order are independent from the other items of the other orders, the expected spread is the same for all orders. Therefore, the index of [j] is dropped from the expression (3.6). The other [j] indices can be dropped as well. The expected gap between order [j-1] and [j] is the expected difference between the position of the first box in order [j] and the last box in order [j-1] Eldemir, et al. (2004).

$$E(gap\ [j] \mid FPR) = E(l_{[j][1]} - l_{[j-1][y]}) = \left(\frac{T}{y+1}\right) \tag{25}$$

The time to sort the entire wave of orders under FPR (TFPR) will be the summation of all the gaps and spreads between orders. It is given as:

$$T(wavetime \mid FPR) = \sum_{[j]=1}^{m}\left[\left[\frac{T.y}{y+1}\right] + \left[\frac{T}{y+1}\right]\right] \\ = \frac{m.T.y}{y+1} \tag{26}$$

Number of the items in an order wave is equal to number of orders times number of items in each order. Namely;

w=m.y (3.9)

If we show the formulas of length of the conveyor is L and circulation time of conveyor is T as below:

$$T\,(wavetime \mid FPR) = \frac{m.T.y}{y+1} = \frac{w^2 l_u}{(y+1)v_c} \tag{27}$$

Thus, sortation time of order wave can be defined also as below:

$$L = w.l_u$$

$$T = \frac{L}{v_c} = \frac{w.l_u}{v_c} \tag{28}$$

Where lu, y and vc parameters values are known. So sortation time of an order wave is only dependent on only the number of the items in a wave (w) parameter. Formula shows that:

When number of the items in an order wave increase, sortation time is increasing too.

4. Integrated Order Picking & Sortation Systems

Determination of the throughput per unit of time give a chance for performance comparisons between warehouse operations and allows us to find more realistic results. Number of the items picked or sorted per unit time can be formulated as below:

For order picking operation:

Number of the items picked per unit of time = (Number of the order wave picked per unit of time)*(Number of the items in an order wave)

If we denote "Number of the items picked per unit of time" as τOrderPicking;

$$\tau_{OrderPicking} = \frac{1}{T_{OrderPicking}}w = \frac{A}{\dfrac{1}{(w+1)}\dfrac{Sl_u}{2.Rows\ .v_p} + T_p} \tag{29}$$

Result of the τOrderPicking shows that when number of the items in an order wave increase number of the items picked per unit time is increasing too.

For order sortation operation:

Number of the items sorted per unit of time = (Number of the order wave sorted per unit of time)*(Number of the items in an order wave)

If we denote "Number of the items sorted per unit of time" as τOrderSortation ;

$$\tau_{OrderSortation} = \frac{1}{T_{OrderSortation}} w = \frac{(y+1)v_c}{wl_u} \quad (30)$$

Result of the τOrderSortation shows that when number of the items in an order wave increase number of the items sorted per unit time is decreasing.

As mentioned in Section 1 order picking and order sortation operations are directly effects the performance of each other. If performance of the order sortation and packaging operation could not be increased by the same ratio, only a bottleneck will be seen between these operations. There will be no meaning of this local improvement for whole system.

Correspondingly, only increasing the performance of order sortation operation does not directly increase the throughput per unit of time. Throughput of the system is equal to throughput performance of the operation which one is bottleneck operation, in the other words operation that has a lower performance.

Therefore determination of the performance for both operations is very important. Proposed analytical model formulations [Formula (29) and fomula (30)] allow for comparing performance of both operations over wave size values.

Only unknown parameter for both Formula (29) and Formula (30) is "w". While performance of order picking operation is a positive function of "w", order sortation performance is a negative function of "w". So determination of optimum point for performance of order picking and sortation operation is only depending on determination of optimum "w" value. Below formula (31) allow us for determining the optimum performance for both operations over "w" value.

$$\max.\left\{\min.\left\{\begin{array}{l}\tau_{OrderSortation} = \dfrac{(y+1)v_c}{wl_u} \\ \tau_{OrderPicking} = \dfrac{A}{\dfrac{1}{(w+1)}\dfrac{Sl_u}{2.Rows.v_p} + T_p}\end{array}\right\}\right\} \quad (31)$$

This is the first proposed analytical model that shows the relationship between order picking and sortation operation and analyze the trade-off between picking and sorting efficiency.

5. Case Study & Empirical Results

We now present an illustrative example to indicate how the proposed analytical model can be used to determine optimum order wave size "w" that makes the performance of both order picking and sortation operation optimum. In particular, the trade-off between the throughput performance of order picking and order sortation is considered as a function of the wave size. The data related to examine illustrative example are reported in Table 4.

Table 4: Main Data for the determination of the optimum waves size

Notation	Definition	Value
w	Number of items within a wave, (unit)	?
A	Number of the aisle in warehouse, (unit)	10
T_p	Picking time of an item from storage bin, (s) *(Walking time for picking is not included)*	20
v_p	Speed of order picker, (m/s)	0,5
v_c	Speed of the conveyors, (m/s)	1
y	number of items within an order, (unit)	10
Rows	Number of racks on top of eachother, (unit)	4
S	Number of total storage bin in warehouse, (unit)	20000
l_u	Width of a storage bin, (m)	0,5

As mentioned in Section 4, there is a trade-off between order picking and sortation operations. By using the analytical formulas (Formula (29) and Formula (30)) with the given data shown in Table 4, graphical result of this trade-off can be seen clearly.

As seen in Figure 2 when order wave size increasing, hourly throughput rate of order picking operation is increasing too but hourly throughput rate of order sortation is decreasing. It is important to determine the optimum wave size that makes the both operations' throughput rate equal. For this example, graphical view of hourly throughput rate of the system is shown in Figure 2. Optimum wave size for given parameters is 40 units. While wave size value is set to 40, hourly throughput rate of the system is being 445 units. 445 units is the maximum throughput rate of the whole system. For any other wave size value system performance will decrease.

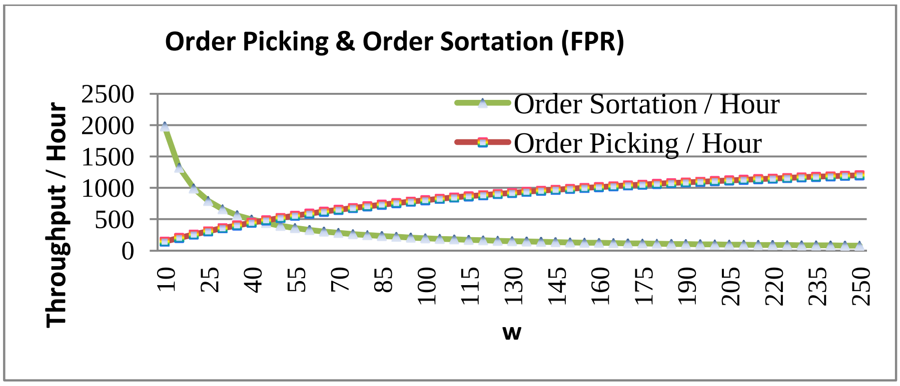

Figure 2: Hourly throughput rate of order picking and order sortation operation

Two different graphs, shown below in Figure 3 and Figure 4, show the hourly throughput rate of the system for different parameter values.

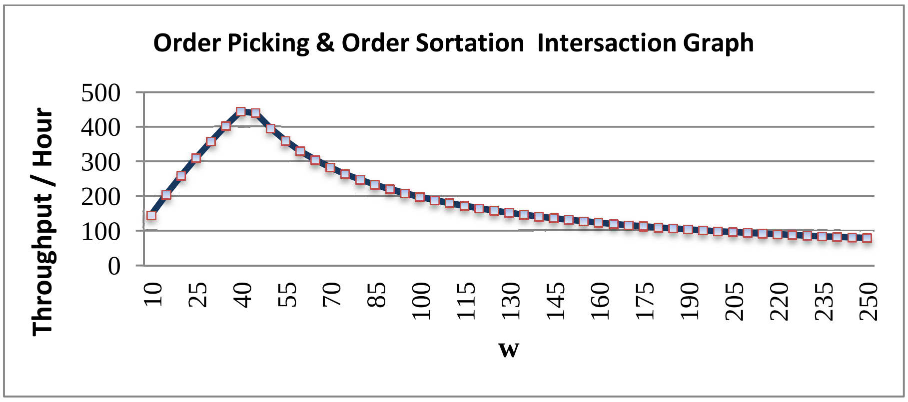

Figure 3: Changes in hourly throughput rate of the system for different wave size.

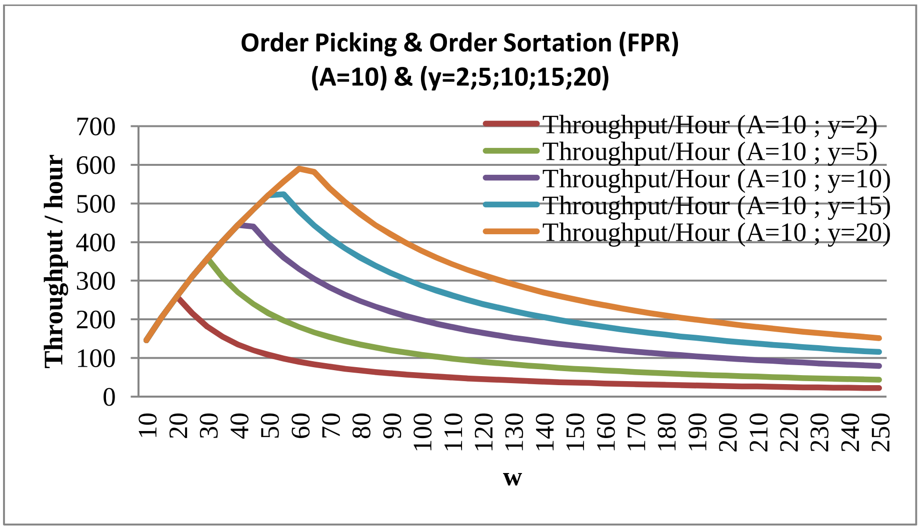

Figure 4: Hourly throughput rate of the system for (A=10) & (y=2;5;10;15;20)

In Figure 4 number of the aisle in warehouse is fixed and throughput rate of the system for different "y" values (number of the items in an order) are analyzed. As shown in Figure 4, while number of the items in an order increases optimum "w" value and throughput rate of the system is increasing too. Optimum numerical values for Figure 4 are shown in Table 5.

Table 5: Optimum "w" and hourly throughput rate of the system for (A=10) & (y=2;5;10;15;20)

	Optimum "w"	Throughput Rate / Hour
(A=10 ; y=2)	20	259
(A=10 ; y=5)	30	358
(A=10 ; y=10)	40	445
(A=10 ; y=15)	55	524
(A=10 ; y=20)	60	590

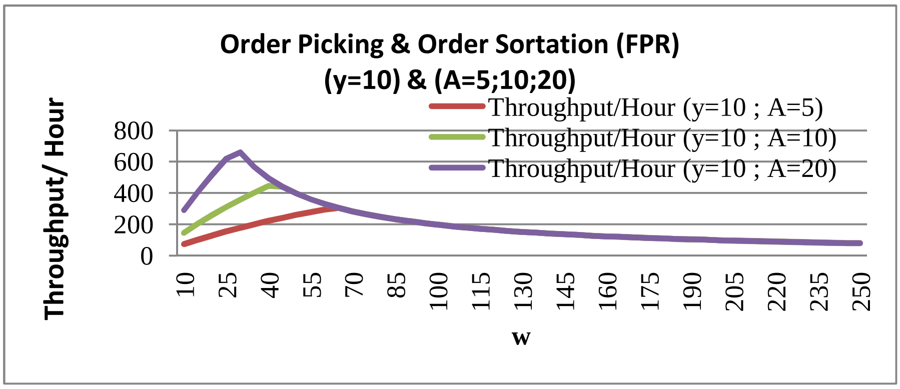

Figure 5: Hourly throughput rate of the system for (y=10) & (A=5;10;15)

In Figure 5 number of the items in an order is fixed and throughput rate of the system for different "A" values (number of the aisle in warehouse) are analyzed. As shown in Figure 5, while number of the aisle in warehouse increase optimum "w" value is decreasing but throughput rate of the system is increasing. Optimum numerical values for Figure 5 are shown in Table 6.

Table 6: Optimum "w" and hourly throughput rate of the system for (y=10) & (A=5;10;20)

	Optimum "w"	Throughput Rate / Hour
(y=10 ; A=5)	65	305
(y=10 ; A=10)	40	445
(y=10 ; A=20)	30	660

6. Discussions and Conclusion

The present study has focused on the integrated pick-and-sort systems. We developed two different analytical models for order picking and order sortation operations that reveal the relationship between these operations. The proposed models consider not only operational parameters but also design parameters. Determining the impact of both operation and design parameters is very important for these types of expressions. Because in warehouse applications design parameters affect the operational parameters and operational parameters also affects the design parameters. So proposed solution method should give answer for the restrictions of both design and operation factors.

This paper propose an analytical solution for the problem that has been mentioned by many papers in literature (related papers are mentioned in Section 1). A model to determine the throughput rate in a unit time for both order picking and order sortation operations has been proposed. Both operations' efficiency has been determined as a function of order wave size. Resultant analytical model give us a chance to solve the trade-off between both order picking and sortation operations by considering the restriction of design and operation factors. So proposed model in this study gives effective and reliable results and can be applied any different warehouses.

The trade-off analysis between order picking and sortation operations refers to a specific sorting system and order picking area layout. That is why it is difficult to show the performance difference of proposed model and real life results. We showed the results via illustrative example. Results of this study can be listed as below:

- Analytical model results showed that when order wave size increase the throughput rate of order picking is increasing but throughput rate of order sortation is decreasing. Figure 2 clearly shows this result.
- Proposed analytical expression gives a chance to calculate the optimum wave size for these two operations. Thus, managers who wish to improve the performance their system can easily determine the bottleneck operation in their system addressing the order picking or order sortation operation.
- Operation and design parameters can be determined by the manager. By using the proposed model, manager can easily observe the changes in total throughput of the system by changing different design and operation parameters. For example when number of aisle in a warehouse increase (number of the items in each order is fixed) throughput rate of the system is increasing but optimum "w" value is decreasing (see Figure 5) or when number of the items in each order increase (aile number is fixed) throughput rate of the system and optimum "w" value is increasing (see Figure 4).

Future research is needed in the area of order picking and sortation systems. They can be listed as below:

- In this paper we have not considered analytical models for One-Many Model and Many-Many Model for sortation operation.
- Other operational and design parameters need to be conducted to see their effects.
- Analytical models for different order picking or storing methods need to be developed.

The proposed observations and results of this study can be used to provide the necessary inside for the development of such models.

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Appendix: Order Statistics

The order statistics of a random sample X_1, ……X_n are the sample values placed in ascending order. The order of the numbers are denoted by $X_{(1)}$, ……$X_{(n)}$. In statistics, the *k*th order statictics value of a randomly selected sample is equel to j*th* smallest value of sample. The order statistics are random variables that sequenced from minimum to maximum like $X_{(1)} \leq X_{(2)} \leq \ldots\ldots X_{(n-1)} \leq X_{(n)}$. Order statistics methods are used for determining the minimum, maximum, median or any ordered value (like j*th* order value) of a randomly selected sample.
Minimum order statistic (or first order statistics) is minimum of the sample [$X_{(1)}$] and maximum order statistic (or last order statistics) is maximum of the sample [$X_{(n)}$] are denoted by;

$X_{(1)}$ = minimum($X_1, X_2, \ldots., X_n$),

$X_{(n)}$ = maximum($X_1, X_2, \ldots., X_n$),

The sample range is the difference between maximum value and minimum value of the sample.

Range(X_1, ……X_n)= $X_{(n)} - X_{(1)}$

When using probability theory to analyze order statistics of random samples from a continuous distribution, the cumulative distribution function is used to reduce the analysis to the case of order statistics of the uniform distribution. (Anon., 2013)

Suppose that $X_1, X_2, \ldots., X_n$ are random variables from a uniform distribution with cumulative distribution function $F_x(x)$ and probability distribution function $f_x(x)$. Asssume that $1 \leq j \leq n$ and probaility distribution function of *j*th order statitistic $X_{(j)}$ is as:

$$f_{X(j)}(x) = \frac{n!}{(j-1)!(n-j)!}[F_x(x)]^{j-1}[1-F_x(x)]^{n-j} f_x(x), \qquad -\infty \leq x \leq \infty \tag{A.1}$$

$X_1, X_2, \ldots., X_n$ values are random variables from a uniform distribution (0, 1). So probability distribution function of uniform distrbution is $f_x(x)=1$ for $x \in (0, 1)$ and $F_x(x)=x$ for $x \in (0, 1)$. Thus, the probability distribution function of the *j*th order statistics is:

$$f_{X(j)}(x) = \frac{n!}{(j-1)!(n-j)!} x^{j-1}[1-x]^{n-j}, \qquad -\infty \leq x \leq \infty \tag{A.2}$$

Expected value of $X_{(j)}$ is:

$$E[X_{(j)}] = \frac{j}{n+1} \tag{A.3}$$

The probability distribution function of the first and last order statistics is:

$$f_1(x) = n[1-F(x)]^{n-1}, \qquad -\infty \leq x \leq \infty \tag{A.4}$$

and

$$f_n(x) = n[F(x)]^{n-1} f(x), \qquad -\infty \leq x \leq \infty$$