Abstract

Banking sector plays a crucial role in economies. As a financial institution that leads the economy and investments, the performance measurement of the bank concerns different segments of the society. The main purpose of the study is to provide decision support for decision makers about the performance of banks by using multi criteria decision making techniques. In accordance with this purpose, the financial performance of twelve commercial banks are evaluated in terms of seventeen financial performance indicators by employing Fuzzy Analytic Hierarchy Process (henceforth Fuzzy AHP) and Fuzzy Technique for Order Preference by Similarity to Ideal Solution (henceforth Fuzzy TOPSIS) methods. The findings of the study show that these two methods rank banks in a similar manner.

Keywords: Fuzzy AHP, Fuzzy TOPSIS, Bank Performance, Multi Criteria Decision Making

JEL Code Classification: C44, G21, L25
1. Introduction

Banking sector plays an important and active role in economies. Any positive or negative events encountered in banking sector affect other sectors through banking activities. Therefore the state of the banking sector is monitored by researchers, academicians and regulatory and supervisory authorities. There is a great deal of studies on the performance measurement of banking sector in the literature. Banking sector is a major driving force of the economy as well. However the financial crises encountered by the countries usually affect primarily the banking sector and then damage other units in the economy. As for recent financial crisis does cause the bankruptcy of many banks around the world. Due to the restructuring of Turkish Banking Sector following local crisis in 2001, the impact of current global crisis on this sector remains relatively limited. Although none of the banks go bankrupt in Turkey, some of the indicators show detriments during the turmoil period. The asset size of the Turkish Banking sector has reached 724.8 billion dollar by the end of 2011. The ratio of the asset size to Gross National Product (GNP) is 94% and the banking sector constitutes 80% of the asset size of financial sector (BRSA, 2011).

In general, performance is understood as the concept that determines the outputs, which arise from a set of intentional and planned activities, in terms of numerical and verbal statements. Therefore the performance of a business system can be defined as the output or operating result at the end of a given period. This output is an indicator of what extend the companies reaches their goals. Consequently, performance can be described as the general evaluation of the all efforts to accomplish their goals by the companies (Akal, 1996).

Performance measurement is a process which shows the efficiency and effectiveness of the activities in terms of quantity. As for performance measure is an indicator of the efficiency and effectiveness of the activities in terms of quantity. A set of these indicators which show the efficiency and effectiveness of the activities is known performance measurement system (Nelly, 2005). Thus, it could not be wrong to say that it is so important to combine performance indicators that are going to be used in performance measurement process for the goals of the company. Because of evaluation and supervision of the efficiency of an activity process depends on the measurement of the performance.

In performance measurement, identifying the performance measurement criteria and their features have crucial role in reaching the goals of company. Atkinson et al., (1997) state that performance criteria should have following characteristics:

- Performance element must be measured independently.
- Measures must be significant and understandable from the viewpoint of user.
- Measures are consistent with the strategic goals of the company.
- Measures are consistent with the competitive strategies of the company.
- Measurement methods must be steady and reliable.
Performance measures must be re-evaluable and changeable if necessary. Measures must be reviewed and accepted by all people from their perspectives.

The purpose of this study is to propose a fuzzy AHP and a fuzzy TOPSIS performance evaluation models for banking sector. Under the increasing uncertainty and competition in global financial markets, measurement of the performance with fuzzy techniques provides clear and reliable information. The measurement of banking performance helps investors in making investment decisions as well as giving information about banks. For this purpose, in this study we use the financial data of 12 commercial banks operating in Turkey. This dataset consists of 17 variables which are categorized in the following topics: Capital Ratios, Assets Quality, Liquidity, Profitability, Income Expenditure Structure, Activity Ratios, Branch Ratios. In the literature, many studies use multiple criteria decision making models to evaluate the financial performance of banking sector. Different from the common literature, we use fuzzy AHP and fuzzy TOPSIS models separately and conclude that these two models give the same results. Accordingly, it is clear that both of these models can be used individually to measure the financial performance of banking sector.

In the second section, we summarize the existing literature on the different methodologies used in the performance measurement in banks. In the third and fourth sections, we explain multiple criteria decision making problems and how to solve these problems via the fuzzy set theory. In the fifth and sixth section, we describe fuzzy AHP and fuzzy TOPSIS methods, respectively. In the seventh section, we present and interpret the empirical findings on the performance evaluation of the Turkish Banking Sector. In the last section, we present conclusion of the study and suggestions for future researches.

2. Literature Review

There is a comprehensive literature on the performance measurement in banking sector. Researchers use various performance measurement approaches in the literature. Ratio analysis is the most widely used method in the performance measurement and comparison (Wen, 2008; Al-Taleb and Al-Shubiri, 2010; Pathak, 2003; Uwuigbe and Fakile, 2012; Aebi et al., 2011; Hernando and Nieto, 2007).

Another method used in the measurement of banking performance is CAMELS ratios. CAMELS consists of six components; capital adequacy, asset quality, management, earnings, liquidity and sensitivity to market list. Dinçer et al., (2011) measure the financial performance of state, private and foreign banks by using CAMELS ratios during the 2008 Global Crisis. Mittal and Dhide (2009) investigate the awareness and perception level of bank employees for CAMEL ratios. Data Envelopment Analysis (DEA) is one of the most commonly used models to measure bank performance (Wang and Huang, 2007; Tsolas, 2010; McEachern and Pradi,
2007; Gregoriou et al., 2008; Şakar, 2006; Özkan et al., 2006; Jemric and Vujcic, 2002; Aikhathlan and Malik, 2010; Sultan et al., 2011). Kumar and Gulati (2010) use DEA method to appraise the efficiency, effectiveness and performance of 27 state banks operating in India and they conclude that high efficiency does not stand for high effectiveness. In addition to these findings they find strong positive relationship between effectiveness and performance. Kantor and Maital (1999) use DEA and Activity Based Cost Accounting method together in their study. The method offer to managers quantitative performance benchmarks in their specific business activity. The study is realized in 250 big banks branches operating in the Middle East. Weiguo and Ming (2008) use DEA method on the 5 biggest American banks and 4 Chinese banks and find that merger and acquisition have more effect on the efficiency of Chinese banks than that of American banks.

Multiple criteria decision making methods are also used by researchers to measure and evaluate the banking performance (Ta and Kar, 2000; Che et al., 2010; Shaverdi et al., 2011). Yurdakul and İç (2004) investigate the credibility values of the companies which may be necessary in the bilateral relationship between production companies and banks by employing AHP approach in Turkey. They obtain a general credibility score based on the financial and nonfinancial scores of the companies. AHP based models are useful due to its ability to combine strategic, operational, financial, non-financial, qualitative and quantitative information together with financial analysis of commercial banks. Seçme et al., (2009) measure the financial and non-financial performance of 5 commercial banks operating in Turkey. They use fuzzy AHP and TOPSIS model to determine the weights of main and sub-criteria and evaluate the performance, respectively.

3. Multi Criteria Decision Making (MCDM)

Decision making problem is a process of selecting the most appropriate one among the all appropriate alternatives. In this kind of problems, it is common to judge for various criteria from among alternatives. In such cases decision maker want to solve multi criteria decision making problem. Multi criteria decision making problem can be stated as follows:

\[ D = \begin{bmatrix} \begin{array}{ccc} C_1 & C_2 & \cdots & C_n \\ A_1 & x_{11} & x_{12} & \cdots & x_{1n} \\ A_2 & x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & x_{m1} & x_{m2} & \cdots & x_{mn} \end{array} \end{bmatrix} \]

\[ W = \{w_1, w_2, \ldots, w_n\} \]

In this matrix, A1,A2,..,Am are possible alternatives which decision makers must select; C1,C2,..,Cn denote alternative criteria of which the performance are measured; xij is the point of the alternative Ai criteria relative to Cj and Wj is the weight of criteria Cj (Chen, 2000).
There are two approaches to solve multi criteria decision making problem. These are multi-objective decision making (MODM) and multi-attribute decision making (MADM) approaches. The goals sometimes contradict with each other. In other words, the optimal solution value for one goal may not be optimal for the others. Material designers try to find common way to reach the optimal solution from among goals. This case reveals infinite solutions referred as pareto optimum solutions. In such models there are decision variables which are constructed as identifying a great or infinite number of decision alternatives in a continuous domain. The best decision is the one of which satisfies the preferences of material designers in accordance with problem restrictions and goals. MADM approach can be used in the selection problems in case decisions have restricted alternatives and performance attributions set. Decision variables may be quantitative or qualitative. When MADM approach is compared with MODM approach, MADM approach includes conservative variable which based on predetermined alternatives and more importantly it does not require certain relationship between input and output variables (Shanian and Savadogo, 2006). Both in production and service systems, optimal decision making for real-world problems can not be evaluated based on simple performance criteria since the real-world is so complex and indefinable. As a consequence multi criteria decision making is used because it provides efficient measurement and evaluation (Zeydan and Çolpan, 2009).

Multi criteria decision making methods provide benefits for evaluators in terms of evaluating various alternatives in different units and from the perspectives of different criteria. This is an important advantage when we compare with the traditional decision support methods in which the all criteria are changed in line with one unit. In addition multi criteria decision making provides an advantage by using quantitative and qualitative evaluation methods together (Bozbura et al., 2007).

Under many circumstances certain data remains incapable in modeling the conditions relating to real-world since human judgment cannot be calculated as certain numerical values and include uncertainty. Using linguistic variables instead of numerical values may be more realistic approach. In the problem, the points and weights of the criteria may be evaluated by linguistic variables (Chen, 2000).

4. Fuzzy Sets

The concept of fuzzy set is introduced firstly by Zadeh (1965). According to Zadeh (1965) a fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one (Zadeh, 1965).

Fuzzy logic which is known as multi-valued logic is used to define and transformation to notable values of human judgment under uncertainty and dynamic system modeling. Fuzzy logic is used in many different fields such as house
tools, robotics, automation, image service, space and defense. Since human judgment and behavior are so complex and not be estimated in certain numerical values, usage of certain values to define service and production system in real-world does not give appropriate results (Zeydan and Çolpan, 2009). For this reason fuzzy set theory with linguistic variables are commonly used to make optimal decisions under the uncertainty environment.

Fuzzy sets were proposed to represent the degree of elements belonging to the specific sets. Instead of using the characteristic function as a mapping function, a fuzzy subset \( \tilde{A} \) of a universal set \( X \) can be defined by its membership function \( \mu_{\tilde{A}}(x) \) as
\[
\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X \},
\]
where \( x \in X \) denotes the elements belonging to the universal set, and \( \mu_{\tilde{A}}(x) : X \rightarrow [0,1] \) (Tzeng and Huang, 2011).

Linguistic variables are the variables whose values are words or sentences in a natural or artificial language. Linguistic variables are stated with effective values such as very high, very good, good, high, normal, very low and very bad (Cheng et al., 2005). Linguistic idioms some like low, middle, high are natural representation of the judgements. These characteristics express the applicability of fuzzy set theory in constitution of preference structure of decision makers. Fuzzy set theory helps to measure uncertainty in concepts via subjective judgements of human being. Further to that, in group decision making, evaluation materialize as a result of evaluators’ view concerning the linguistic variables and this evaluation should be performed under the uncertain and fuzzy environment (Saghafian and Hejazi, 2005).

In the literature, it is seen that the most widely used fuzzy numbers are triangular and trapezoidal ones. Especially the fuzzy numbers, which we use in this study, is the most preferred one due to easy of calculation. The triangular fuzzy numbers can be denoted by \((l, m, u)\). Its membership function \( \mu_{\tilde{A}}(x) : R \rightarrow [0,1] \) is equal to
\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x - l}{m - l}, & x \in [l, m] \\
\frac{x - u}{m - u}, & x \in [m, u] \\
0, & \text{otherwise}
\end{cases}
\]
(1)

Where \( l \leq m \leq u \), \( l \) and \( u \) stand for the lower and upper value of the support of \( M \) respectively, and \( m \) for the modal value. Consider two triangular fuzzy numbers \( M_1 \) and \( M_2 \), \( M_1 = (l_1, m_1, u_1) \) and \( M_2 = (l_2, m_2, u_2) \). Their operational laws are as follows:
Fuzzy Performance Evaluation with AHP and Topsis Methods...

\[
(l_1, m_1, u_1) \ominus (l_2, m_2, u_2)
\]
\[
= (l_1 + l_2, m_1 + m_2, u_1 + u_2).
\]
\[
(l_1, m_1, u_1) \otimes (l_2, m_2, u_2)
\]
\[
= (l_1, m_1, u_1) = (l_1, m_1, u_1).
\]

\[\lambda > 0, \lambda \in R\]

\[
(l_1, m_1, u_1)^{-1} = (1/l_1, 1/m_1, 1/u_1) \quad (Chang, 1996)
\]

TOPSIS and AHP are the most commonly used multi decision making methods in the literature. The main shortcomings of these methods are the impact of the uncertainty on decision criteria. However, using fuzzy set theory can overcome this problem.

5. Fuzzy AHP Method

AHP method is one of the multi criteria decision making approaches. AHP is a general measurement theory. AHP is the most comprehensive application which is used in multi criteria decision making, planning, resource allocation and solution of the problems (Saaty and Vargas, 2000). AHP is firstly introduced by Myres and Alpert in 1968 and developed by Saaty in 1977 as a model and then used in the solution of complex decision problems. AHP helps to identify the most appropriate alternative by providing to form hierarchical structure that include target, main criteria, sub-criteria and alternatives and evaluating the many alternatives in terms of many quantitative and qualitative criteria, together (Girginer, 2008).

AHP is a basic approach in decision making. AHP is designed with the intent of selecting the best from among the different alternatives, which are evaluated by using a few criteria, in a both rational and heuristic manner. In this process, decision maker presents simple pair wise comparison judgments. These judgments are then used in the development of all priorities to rank the alternatives. In the hierarchical structure of decision problem, the simple figure, which has three levels, is used. The goal of the decision problem is at the top level. Decision criteria and decision alternatives to be evaluated take part at the second and third levels, respectively (Saaty and Vargas, 2000).

AHP method which is introduced by Saaty simplifies the solution of the multi-attribute, multi-person, multi-period structural problems. In spite of the fact that the goal of AHP is to evaluate expert information, it cannot give an answer to uncertainty in the way of human thinking. Therefore, fuzzy AHP is suggested to solve this kind of problem (Büyüközkan and Çiftçi, 2012). Decision makers use natural linguistic emphasis as well as certain numbers to evaluate criteria and alternatives. Fuzzy AHP resemble impressively human thought and perception. For this reason many researchers use fuzzy AHP method in the literature (Heo et al., 2007).
In the literature, fuzzy AHP method is frequently used in the selection and evaluation of the supplier (Sun, 2010; Xia and Wu, 2007; Chamodrakas and Martakos, 2010; Krishnendu et al., 2012; Kılıncı and Önal, 2011; Lee, 2009). In addition to the selection and evaluation of supplier, fuzzy AHP is used in different fields by following researchers: Jyoti and Deshmukh (2008) in the performance evaluation of national R&D companies; Aydın and Arslan (2010) in the optimal region selection of hospitals; Li and Chen (2009) to evaluate the architectural design service; Enea and Piazza (2004) for project selection decisions; Perçin (2008) to evaluate the benefits of information sharing decision in supply chain; Mohaghar et al., (2012) to select the market strategies; Güngör et al., (2009) in personnel selection problems; Çelik et al., (2009) to select shipping registry; Büyüközkan et al., (2011) for strategic analysis of service quality in health sector; Dağdeviren et al., (2009) in weapons selection decisions; Chou Sun and Yen (2012) to evaluate human resources selection criteria.

In our study, we use Chang’s (1996, 1999) extent analysis method to compare and evaluate the bank’s performance. Chang’s (1996, 1999) extent analysis can be described as follows:

\[ X = \{x_1, x_2, \ldots, x_n\} \] be an object set, and \[ U = \{u_1, u_2, \ldots, u_m\} \] be a goal set.

In extent analysis, each subject is taken in order to accomplish a goal. By this way each subject has m extent analysis value as following:

\[ M_{i\mu}, M_{i\nu}, \ldots, M_{i\omega}, \quad i = 1, 2, \ldots, n \] (3)

Where all the \[ M_{i\mu} \] \( (j = 1, 2, \ldots, m) \) are triangular fuzzy numbers. Fuzzy synthetic extent value for i-th subject can be defined as:

\[ S_i = \sum_{i=1}^{n} M_{i\mu} \odot \left[ \sum_{i=1}^{n} \sum_{i=1}^{m} M_{i\mu} \right]^{-1} \] (4)

Here \( S_i \) denotes synthesis value of i-th goal. To calculate \( \sum_{i=1}^{n} M_{i\mu} \) value, m extent analysis values are calculated by using fuzzy addition and then a matrix is obtained.

\[ \sum_{i=1}^{n} M_{i\mu} = \left\{ \sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right\} \] (5)

From this point of view, to calculate \( \left[ \sum_{i=1}^{n} \sum_{i=1}^{m} M_{i\mu} \right]^{-1} M_{i\mu} \) \( (j = 1, 2, \ldots, m) \) values and then the transpose of this vector are calculated as:

\[ \left[ \sum_{i=1}^{n} \sum_{i=1}^{m} M_{i\mu} \right]^{-1} = \left[ \frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right] \]
2. Step: While \( \tilde{M}_1 \) and \( \tilde{M}_2 \) are two triangular fuzzy numbers, the probability degree of following equation
\[
V(\tilde{M}_2 \geq \tilde{M}_1) = \sup_{x \in \mathbb{R}} \min \left( \mu_{\tilde{M}_2}(x), \mu_{\tilde{M}_1}(x) \right)
\]
\( \tilde{M}_1 \geq \tilde{M}_1 \) is defined as:
\[
V(\tilde{M}_2 \geq \tilde{M}_1) = \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, \text{ others}
\]
Between two fuzzy numbers like \( \tilde{M}_1 \) and \( \tilde{M}_2 \), the probability of the case in which \( \tilde{M}_2 \) is greater than \( \tilde{M}_1 \) and other cases can be shown as:
\[
V(\tilde{M}_2 \geq \tilde{M}_1) = \begin{cases} 
1, & \text{if } m_2 \geq m_1 \\
0, & \text{if } l_1 \geq u_2 \\
\frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{others}
\end{cases}
\]
Where \( d \) is the ordinate of the highest intersection point \( D \) between \( \mu_{\tilde{M}_1} \) and \( \mu_{\tilde{M}_2} \).

![Fig.1: Intersection point D between \( \mu_{\tilde{M}_1} \) and \( \mu_{\tilde{M}_2} \)](image)

3. Step: The degree possibility for a convex fuzzy numbers to be greater than \( k \) convex fuzzy numbers \( M_1, M_2, \ldots, M_k \) can be defined by
\[
V(M \geq M_1, M_2, \ldots, M_k) = \min \left\{ V(M_1 \geq M_1), \ldots, V(M_k \geq M_k) \right\}
\]
Assume that
\[
d'(A_i) = \min V(S_i \geq S_i)
\]
for \( k = 1, 2, \ldots, n; k \neq i \) then the weight vector is given by
\[
W = (d'(A_1), d'(A_2), \ldots, d'(A_n))
\]
Where \( A_i (i = 1, 2, \ldots, n) \) are \( n \) elements.
4. Step: Via normalization, we get the normalized weight vectors
\[ w = (d(A_1), d(A_2), \ldots, d(A_n))^T \] (12)
Where \( w \) is a nonfuzzy number.

6. Fuzzy TOPSIS Method

TOPSIS, which is firstly introduced by Hwang and Yoon, is a multi criteria decision making method widely used in multi criteria decision making problems. TOPSIS method based on the fact that selected alternative is the shortest distance to positive ideal solution and the longest distance to negative ideal solution (Chen, 2000). TOPSIS method do not solely assert the distance of selection alternative to positive and negative ideal solution and also present ideal and nonideal solutions (Wang et al., 2009).

TOPSIS is widely used in different fields of multi criteria group decision making. The first reason for this is that different from AHP or simple weighted summation methods, TOPSIS based on the opinion that it presents the best appropriate result as the shortest distance to positive ideal solution or longest distance to negative ideal solution. Second, TOPSIS is a method that is heuristic, understandable and simple. Third, TOPSIS has some advantages compared to other methods:

- The performance of the method is partially affected by the number of alternatives and powered by the increasing number of alternative and criteria in rank differences.
- The order of alternatives may change when non-optimal alternative is entered (Bottani and Rizzi, 2006).

In addition with the advantages of the method, under uncertainty and changing environment fuzzy TOPSIS method is used to prevent unfavorable conditions arising from the changes in decision criteria. Fuzzy TOPSIS method overcomes the some shortcomings of TOPSIS method. These are:

a) The need to assign initial weights for each criteria
b) When the fuzzy numbers are 1 and 0, it is assumed that these are positive and negative ideal solution, respectively. When weights and rated values are so large, the distance between criteria and fuzzy positive and negative ideal solutions increase.
c) The results sometime do not comply with the main idea; in this case the best solution is the shortest distance to ideal solution and the longest distance to negative ideal solution (Wang et al., 2009).

Fuzzy TOPSIS method is used in different fields in the literature. Mangır and Erdoğan (2011) use fuzzy TOPSIS model to compare the macroeconomic performance (Economic Growth, Inflation Rate, Unemployment Rate, Current Account Balance, Budget Balanced Rate) in six countries during the global crisis (Italy, Greece, Spanish, Portugal, Ireland and Turkey). Fuzzy TOPSIS method is also
Fuzzy Performance Evaluation with AHP and Topsis Methods...


Fuzzy TOPSIS method has following steps:

Step 1: To identify evaluation criteria by forming a group of decision makers.

Step 2: To select appropriate linguistic variables and linguistic score for alternatives according to the weights of criteria.

Step 3: The weight of criteria are aggregated to get the fuzzy weight $W_j$, of $C_j$ criteria. In accordance with the opinion of decision makers, aggregated fuzzy numbers, $x_{ij}$ are obtained for $A_i$ alternative under $C_j$ criteria. This is calculated as:

$$
\bar{X}_{ij} = \frac{1}{k} \left( x_{ij,1} \oplus x_{ij,2} \oplus \ldots \oplus x_{ij,k} \right)
$$

$$
\bar{W}_j = \frac{1}{k} \left( w_{j,1} \oplus w_{j,2} \oplus \ldots \oplus w_{j,k} \right)
$$

Step 4: Fuzzy decision matrix and normalized decision matrix are created. The linguistic variables in fuzzy decision matrix are defined as triangular fuzzy numbers $\{ \bar{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}) \}$ and $\bar{w}_{ij} = (w_{j,1}, w_{j,2}, w_{j,3})$. The normalization of fuzzy decision matrix in terms of benefit (Benefit-B) and cost criteria (Cost-C) is realized as:

$$
\bar{T}_{ij} = \left( \frac{a_{ij}}{c_{ij}}, \frac{b_{ij}}{c_{ij}}, \frac{c_{ij}}{c_{ij}} \right), \ j \in B
$$

$$
\bar{T}_{ij} = \left( \frac{a_{ij}}{c_{ij}}, \frac{a_{ij}}{B_{ij}}, \frac{a_{ij}}{c_{ij}} \right), \ j \in C
$$

$$
c^* = \max_j c_{ij}, \ \text{if} \ j \in B
$$

$$
\bar{a}_{ij} = \min_j a_{ij}, \ \text{if} \ j \in C
$$

Step 5: Weighted normalized fuzzy decision matrix is constructed as

$$
\bar{V}_{ij} = \bar{T}_{ij} \otimes \bar{W}_j
$$

Step 6: Fuzzy positive ideal solution and fuzzy negative ideal solution set is formed as

$$
\bar{V}^*_{ij} = (1,1,1)
$$

$$
\bar{V}^-_{ij} = (0,0,0)
$$
Step 7: The distance of each alternative to fuzzy positive ideal solution set and fuzzy negative ideal solution set are calculated by using vertex method. Vertex method is used to calculate the distance between fuzzy numbers. The distance between two triangular numbers as \( \tilde{m} = (m_1, m_2, m_3) \) and \( \tilde{n} = (n_1, n_2, n_3) \) is calculated by using vertex method as following:

\[
\begin{align*}
    d(\tilde{m}, \tilde{n}) &= \frac{1}{\sqrt{3}} \left[ (m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 \right] \\
\end{align*}
\]

Step 8: The closeness coefficient of each alternative is calculated as following:

\[
CC_i = \frac{d_i^*}{d_i^* \oplus d_i^*}, \quad i = 1, 2, ..., m
\]

Step 9: The ranking of the all alternatives are determined according to the closeness coefficient (Chen, 2000).

7. Empirical Study

Banks are the most important financial institutions that direct the economy and investment at the countries. Banks’ financial performances are important indicators for decision makers in investment decision making process. In this study, we compare the financial performance of 12 commercial banks operating in Turkey and traded on Istanbul Stock Exchange by employing Fuzzy AHP and Fuzzy TOPSIS. There are 31 commercial banks in Turkey. The assets size of 12 commercial banks which are used in this study constitutes 78% of the asset size of commercial banks (BRSA, 2011; BRSA and BAT data query system). The financial data used in this study which is belonging to year 2010 is obtained from The Banks Association of Turkey (www.tbb.org.tr).

Uncertainty that appeared on the year 2008 which belonged to the global crisis has negatively affected banking performances. Usage of fuzzy multi criteria decision making techniques may provide better results under uncertain market conditions. Since the dataset covers the financial ratios of the banks for the year 2010 which correspond to after 2008 global crisis, the dataset is significant in terms of giving information about the financial performance of banks after the global crisis.

The financial performance of banks is measured with 17 financial ratios classified under 7 mean categories. Descriptive statistics for the financial ratios of banks are shown in Table 1.

7.1. Performance Measurement via Fuzzy AHP

Firstly, we measure the performance of banks by employing Fuzzy AHP method. In Fuzzy AHP method, in accordance with expert opinion, the comparison matrix regarding criteria and alternatives is constructed as triangular fuzzy numbers. Table 2 presents the linguistic assessment variables and its corresponding scale in terms of triangular fuzzy numbers.
### Table 1: Descriptive Statistics for the Financial Ratios

<table>
<thead>
<tr>
<th>Ratios</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Min. Value</th>
<th>Max. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capital Ratios %</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shareholders' Equity / (Amount Subject to Credit Risk + Market Risk + Operational Risk)</td>
<td>16.66</td>
<td>2.20</td>
<td>14</td>
<td>20.60</td>
</tr>
<tr>
<td>Shareholders' Equity / Total Assets</td>
<td>12.73</td>
<td>2.62</td>
<td>9.50</td>
<td>19.30</td>
</tr>
<tr>
<td><strong>Assets Quality, %</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Loans and Receivables / Total Deposits</td>
<td>96.71</td>
<td>16.66</td>
<td>72.80</td>
<td>132.90</td>
</tr>
<tr>
<td>Loans under follow-up (gross) / Total Loans and Receivables</td>
<td>4.43</td>
<td>1.43</td>
<td>2.40</td>
<td>7.10</td>
</tr>
<tr>
<td><strong>Liquidity, %</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquid Assets / Short-term Liabilities</td>
<td>46.74</td>
<td>17.33</td>
<td>20.20</td>
<td>79.30</td>
</tr>
<tr>
<td><strong>Profitability, %</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Profit (Losses) / Total Assets</td>
<td>1.88</td>
<td>0.71</td>
<td>0.60</td>
<td>2.80</td>
</tr>
<tr>
<td>Net Profit (Losses) / Total Shareholders' Equity</td>
<td>15.28</td>
<td>6.32</td>
<td>2.90</td>
<td>27</td>
</tr>
<tr>
<td><strong>Income Expenditure Structure, %</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Interest Income After Specific Provisions / Total Assets</td>
<td>3.58</td>
<td>0.78</td>
<td>2.60</td>
<td>5.40</td>
</tr>
<tr>
<td>Non-Interest Income (Net) / Total Assets</td>
<td>2.18</td>
<td>0.62</td>
<td>1.40</td>
<td>3.40</td>
</tr>
<tr>
<td>Provision For Loan or Other Receivables Losses / Total Assets</td>
<td>0.99</td>
<td>0.38</td>
<td>0.5</td>
<td>1.70</td>
</tr>
<tr>
<td>Interest Income / Interest Expense</td>
<td>211.99</td>
<td>26.85</td>
<td>186.6</td>
<td>280.60</td>
</tr>
<tr>
<td>Total Income / Total Expense</td>
<td>148.11</td>
<td>16.02</td>
<td>122.70</td>
<td>170.50</td>
</tr>
<tr>
<td><strong>Branch Ratios, Million TRY</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Loans and Receivables / No. of Branches</td>
<td>52.92</td>
<td>14.84</td>
<td>27</td>
<td>75</td>
</tr>
<tr>
<td>Net Income / No. of Branches</td>
<td>1.92</td>
<td>1.16</td>
<td>0.00</td>
<td>4.00</td>
</tr>
<tr>
<td><strong>Activity Ratios</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Personnel Expenses + Reserve for Employee Termination Benefit) / Total Assets</td>
<td>1.45</td>
<td>0.48</td>
<td>0.80</td>
<td>2.20</td>
</tr>
<tr>
<td>(Personnel Expenses + Reserve for Employee Termination Benefit) / Number of Personnel (Thousand TRY)</td>
<td>62.55</td>
<td>6.77</td>
<td>50.80</td>
<td>70.00</td>
</tr>
<tr>
<td>Net Operating Income(Loss) / Total Assets</td>
<td>2.30</td>
<td>0.92</td>
<td>0.80</td>
<td>3.40</td>
</tr>
</tbody>
</table>

### Table 2: The Linguistic Assessment Variables and its Corresponding Scale in terms of Triangular Fuzzy Numbers

<table>
<thead>
<tr>
<th>Linguistic Variable</th>
<th>Triangular Fuzzy Scale</th>
<th>Triangular Fuzzy Reciprocal Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal</td>
<td>(1,1,1)</td>
<td>(1/1, 1/1, 1/1)</td>
</tr>
<tr>
<td>Moderate</td>
<td>(2,3,4)</td>
<td>(1/4, 1/3, 1/2)</td>
</tr>
<tr>
<td>Strong</td>
<td>(4,5,6)</td>
<td>(1/6, 1/5, 1/4)</td>
</tr>
<tr>
<td>Very Strong</td>
<td>(6,7,8)</td>
<td>(1/8, 1/7, 1/6)</td>
</tr>
<tr>
<td>Extremely Preferred</td>
<td>(8,9,9)</td>
<td>(1/9, 1/9, 1/8)</td>
</tr>
</tbody>
</table>
The consistency of pairwise comparison matrix which is stated with linguistic variables and formed in line with common opinion of experts is evaluated by using Kwong and Bai (2003) approach. In this approach, a triangular number as $M = (l, m, u)$ is transformed to a non-fuzzy number by using following notation, $\frac{4m + l + u}{6}$ and then the consistency control may be done as in AHP method. In our study, the consistency ratio is less than 0.10 in all matrices, which are constructed concerning criteria and alternatives, and so all matrices is found to be consistent. We use the study of Alonso and Lamata (2006) to identify the random index coefficients in the calculation of consistency.

Firstly, synthetic values are calculated according to Chang’s (1996) extent analysis method with equation 4 by using the data in pairwise comparison matrix which is created in line with experts’ opinion:

\[
S_1 = (53.65, 76) \odot \left( \begin{array}{ccc}
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} \\
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} \\
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} 
\end{array} \right) = (0.0738, 0.1079, 0.1562)
\]

\[
S_2 = (30.25, 37.6667, 45.5) \odot \left( \begin{array}{ccc}
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} \\
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} \\
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} 
\end{array} \right) = (0.0421, 0.0625, 0.0935)
\]

\[
S_3 = (15.0833, 19.6667, 24.75) \odot \left( \begin{array}{ccc}
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} \\
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} \\
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} 
\end{array} \right) = (0.0210, 0.0327, 0.0509)
\]

\[
S_4 = (30.25, 37.6667, 45.5) \odot \left( \begin{array}{ccc}
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} \\
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} \\
\frac{1}{7182083} & \frac{1}{6023460} & \frac{1}{4865556} 
\end{array} \right) = (0.0421, 0.0625, 0.0935)
\]

After acquiring synthesis values, the comparison of the fuzzy numbers are done with Equation 8 as following:

\[
V(S_1 \geq S_2) = 1
\]
\[
V(S_1 \geq S_3) = 1
\]
\[
V(S_2 \geq S_1) = 0.3029
\]
\[
V(S_2 \geq S_3) = 1
\]
\[
V(S_3 \geq S_1) = 0.3029
\]
\[
V(S_3 \geq S_2) = 1
\]
\[
V(S_3 \geq S_4) = 1
\]
\[
V(S_4 \geq S_3) = 0.3029
\]
\[
V(S_{17} \geq S_3) = 1
\]
\[
V(S_{17} \geq S_4) = 1
\]
\[
V(S_{17} \geq S_5) = 0.3029
\]
\[
V(S_{17} \geq S_6) = 1
\]
After this stage, the priority values of criteria are calculated by using Equation 10;

\[
d'(K_1) = \min (1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1, 1.1) = 1
\]

\[
d'(K_2) = \min (0.3029, 0.10, 0.3029, 0.10, 0.3029, 0.1, 0.1, 0.1, 0.1, 0.3029) = 0.3029
\]

\[
d'(K_3) = \min (0.02265, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1) = 0
\]

\[
d'(K_r) = \min (0.3029, 0.10, 0.3029, 0.1, 0.3029, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1) = 0.3029
\]

The weight vector is as following:

\[
W = (0.3029, 0.10, 0.3029, 0.1, 0.3029, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1)
\]

The values in this vector are normalized to find the priority values of criteria;

\[
W = (0.1535, 0.0465, 0.1535, 0.0, 0.1535, 0.0, 0.0465, 0.0465, 0.0465, 0.0, 0.1535, 0.0, 0.0465)
\]

After applying the same stages for all alternatives, the table which shows decision criteria regarding criteria and alternatives is as following:

As a result of Fuzzy AHP method, we rank the performance scores based on the financial data of 12 commercial banks operating in Turkey. As seen in Table 3, the first 5 banks which have the highest performance scores are C, H, A, J and I banks respectively.

Table 3: Performance Rating of Banks via Fuzzy AHP Method

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
<th>C12</th>
<th>C13</th>
<th>C14</th>
<th>C15</th>
<th>C16</th>
<th>C17</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Bank</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>B Bank</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>C Bank</td>
<td>0.117</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>D Bank</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E Bank</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>F Bank</td>
<td>0.195</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>G Bank</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>H Bank</td>
<td>0.195</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>I Bank</td>
<td>0.195</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>J Bank</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>K Bank</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
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</tr>
<tr>
<td>L Bank</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

7.2. Performance Measurement via Fuzzy TOPSIS

Fuzzy TOPSIS is one of the methods which are used for decision making in fuzzy environment. After applying Fuzzy AHP method, we also apply Fuzzy TOPSIS method suggested by Chen (2000) in measuring bank performance to obtain comparable results. Linguistic variables used in evaluating the criteria are displayed in Table 4.
Table 4: Linguistic Variables Used in Evaluating the Criteria

<table>
<thead>
<tr>
<th>Linguistic Variables</th>
<th>Triangular Fuzzy Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low</td>
<td>(0, 0, 0.1)</td>
</tr>
<tr>
<td>Low</td>
<td>(0, 0.1, 0.3)</td>
</tr>
<tr>
<td>Moderate-Low</td>
<td>(0.1, 0.3, 0.5)</td>
</tr>
<tr>
<td>Moderate</td>
<td>(0.3, 0.5, 0.7)</td>
</tr>
<tr>
<td>Moderate-High</td>
<td>(0.5, 0.7, 0.9)</td>
</tr>
<tr>
<td>High</td>
<td>(0.7, 0.9, 1.0)</td>
</tr>
<tr>
<td>Very High</td>
<td>(0.9, 1.0, 1.0)</td>
</tr>
</tbody>
</table>

As for linguistic variables used in evaluating the alternatives are displayed in Table 5.

Table 5: Linguistic Variables Used in Evaluating the Alternatives

<table>
<thead>
<tr>
<th>Linguistic Variables</th>
<th>Triangular Fuzzy Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Weak</td>
<td>(0, 0, 1)</td>
</tr>
<tr>
<td>Weak</td>
<td>(0, 1, 3)</td>
</tr>
<tr>
<td>Moderate-Weak</td>
<td>(1, 3, 5)</td>
</tr>
<tr>
<td>Moderate</td>
<td>(3, 5, 7)</td>
</tr>
<tr>
<td>Moderate- Good</td>
<td>(5, 7, 9)</td>
</tr>
<tr>
<td>Good</td>
<td>(7, 9, 10)</td>
</tr>
<tr>
<td>Very Good</td>
<td>(9, 10, 10)</td>
</tr>
</tbody>
</table>

Fuzzy decision matrix formed in line with the opinion of decision makers. Following, normalized decision matrix is constructed and then weighted normalized fuzzy decision matrix is formed by multiplying each value in normalized decision matrix with the relating weight of the criteria by using Equation 15 and 17.

After generating weighted normalized fuzzy decision matrix, we calculate the distance of each alternative to fuzzy positive ideal solution and fuzzy negative ideal solution for all criteria by using Equation 18, 19, 20. The distance of alternatives to the positive and negative ideal solutions for all criteria are given in Table 6 and Table 7.
Table 6: The Distance to Fuzzy Positive Ideal Solution for Each Criteria

The banks are ranked according to the relative distance values of alternatives (CCI). The first 5 banks which have the highest performance scores according to Fuzzy TOPSIS method are H, C, J, A and I banks, respectively. In conclusion, despite the small differences in scores, the first 5 banks are same in Fuzzy AHP and Fuzzy TOPSIS methods.

8. Conclusion
Motivated by the following statement “if it's not measured, it's not managed”, measuring the performance of banks, which play an important role in economy, accurately is crucial. Uncertainty and complexity in the current global markets, in which trade volume and the speed of information and communication increase, are the biggest obstacles in measuring the performance accurately. Traditional performance measurement methods are not sufficient under the uncertainty and complexity. However, the fuzzy multi criteria decision making approaches are recently used to overcome this problem.

In this study, we use two different fuzzy multi criteria decision making approaches which serve the same purpose. While Fuzzy AHP method measure the performance in a hierarchical structure by use of pairwise comparison matrix, Fuzzy TOPSIS method measure the performance by use of the distance to fuzzy negative and fuzzy positive ideal principle. These two methods are commonly used in the literature relative to the other methodologies.

As a result, we observe that the performance ranking of the banks is similar for both methods. The study will contribute to literature because it does present the general outlook of Turkish banking sector after the 2008 global crisis and beneficial to investors who need to make investment decisions. For future studies, the performance of different fuzzy multi criteria decision making approaches can be compared. Similarly, different methods can be compared by using skewed fuzzy numbers to decrease uncertainty.

References


